Mesonic states in the generalised Nambu-Jona-Lasinio theories

A. V. Nefediev* and J. E. F. T. Ribeiro†

*Institute of Theoretical and Experimental Physics, 117218, B.Cheremushkinskaya 25, Moscow, Russia
†Centro de Física das Interacções Fundamentais (CFIF), Departamento de Física, Instituto Superior Técnico, Av. Rovisco Pais, P-1049-001 Lisboa, Portugal

Abstract. For any Nambu-Jona-Lasinio model of QCD with arbitrary nonlocal, instantaneous, quark current-current confining kernels, we use a generalised Bogoliubov technique to go beyond BCS level (in the large-N_C limit) so as to explicitly build quark-antiquark compound operators for creating/annihilating mesons. In the Hamiltonian approach, the mesonic bound-state equations appear (from the generalised Bogoliubov transformation) as mass-gap-like equations which, in turn, ensure the absence, in the Hamiltonian, of mesonic Bogoliubov anomalous terms. We go further to demonstrate the one-to-one correspondence between Hamiltonian and Bethe-Salpeter approaches to non-local NJL-type models for QCD and give the corresponding "dictionary" necessary to "translate" the amplitudes built using the graphical Feynman rules to the terms of the Hamiltonian, and vice versa. We comment on the problem of multiple vacua existence in such type of models and argue that mesonic states in the theory should be prescribed to have an extra index — the index of the replica in which they are created. Then the completely diagonalised Hamiltonian should contain a sum over this new index. The method is proved to be general and valid for any instantaneous quark kernel.

We study generalised Nambu-Jona-Lasinio models which are expected to mimic the most important low-energy properties of QCD [1, 2]. The Hamiltonian reads:

$$\hat{H}=\int d^3x\bar{\psi}(\vec{x},t)\left(-i\vec{\gamma}\cdot\vec{\bigtriangledown}+m\right)\psi(\vec{x},t)+\frac{1}{2}\int d^3xd^3y\,J^a_\mu(\vec{x},t)K^{ab}_{\mu\nu}(\vec{x}-\vec{y})J^b_\nu(\vec{y},t), \quad (1)$$

and contains the interaction of the quark currents $J^a_\mu(\vec{x},t)=\bar{\psi}(\vec{x},t)\gamma_\mu\frac{\lambda^a}{2}\psi(\vec{x},t)$, parameterised through the instantaneous quark kernel, $K^{ab}_{\mu\nu}(\vec{x}-\vec{y})=g_{\mu0}g_{\nu0}\delta^{ab}V_0(|\vec{x}-\vec{y}|)$, with a power-like confining potential $V_0(|\vec{x}|)=K_0^{\alpha+1}|\vec{x}|^\alpha$. The standard approach to the theories (1) is the Bogoliubov-Valatin transformation defined by the chiral angle φ_p [2],

$$\left\{\begin{array}{rcl} u(\vec{p}) & = & \frac{1}{\sqrt{2}}\left[\sqrt{1+\sin\varphi_p}+\sqrt{1-\sin\varphi_p}\,(\vec{\alpha}\hat{\vec{p}})\right]u(0),\\ v(-\vec{p}) & = & \frac{1}{\sqrt{2}}\left[\sqrt{1+\sin\varphi_p}-\sqrt{1-\sin\varphi_p}\,(\vec{\alpha}\hat{\vec{p}})\right]v(0). \end{array}\right. \quad (2)$$

Anomalous terms in the Hamiltonian vanish if φ_p obeys the mass-gap equation,

$$m\cos\varphi_p-p\sin\varphi_p=\frac{C_F}{2}\int\frac{d^3k}{(2\pi)^3}V_0(\vec{p}-\vec{k})\left[\sin\varphi_k\cos\varphi_p-(\hat{\vec{p}}\hat{\vec{k}})\cos\varphi_k\sin\varphi_p\right]. \quad (3)$$

Eq. (3) is subject to numerical studies. As soon as a nontrivial solution $\varphi_0(p)$ to the mass-gap equation is built, it defines the vacuum of the theory with spontaneously broken chiral symmetry, which is energetically preferable as compared to the unbroken phase. The terms in (1) quartic in quark operators contain the suppressing factor of $1/\sqrt{N_C}$ and thus the Hamiltonian of the theory is diagonalised in the quark sector (BCS level).

To proceed beyond BCS level and reformulate the theory in terms of colourless mesonic states, we notice that only operators creating/annihilating $q\bar{q}$ pairs are allowed:

$$\hat{M}_{ss'}(\vec{p},\vec{p}) = \frac{1}{\sqrt{N_C}}\hat{d}_{\alpha s}(-\vec{p})\hat{b}_{\alpha s'}(\vec{p}) = \sum_{\nu}[\kappa_\nu(\hat{\vec{p}})]_{ss'}\sum_{n}[\hat{m}_{n\nu}\varphi^{+}_{n\nu}(p)+\hat{m}^{\dagger}_{n\nu}\varphi^{-}_{n\nu}(p)], \quad (4)$$

all other operators, like $\hat{b}^\dagger\hat{b}$ and $\hat{d}^\dagger\hat{d}$, being suppressed by N_C [3, 4]. In (4) we consider the $q\bar{q}$ pair at rest and separate the spin-angular and the radial parts of the operator $\hat{M}$. The complete set $\{\kappa_\nu\}$ is chosen to be the J^{PC} set of states, which are known to diagonalise the Hamiltonian of strong interactions. We also perform a second, generalised, Bogoliubov-like transformation and introduced the mesonic creation/annihilation operators $\hat{m}^{\dagger}_{n\nu}/\hat{m}_{n\nu}$. The Bogoliubov amplitudes $\varphi^{\pm}_{n\nu}$ obey the normalisation condition which follows immediately from the commutation relation for the mesonic operators:

$$[\hat{m}_{n\nu},\,\hat{m}^{\dagger}_{m\nu}] = \int\frac{p^2dp}{(2\pi)^3}\left[\varphi^{+}_{n\nu}(p)\varphi^{+}_{m\nu}(p)-\varphi^{-}_{n\nu}(p)\varphi^{-}_{m\nu}(p)\right] = \delta_{nm}. \quad (5)$$

Meanwhile, $\varphi^{\pm}_{n\nu}$ also play the role of the two components of the mesonic wave function responsible for the forward and backward in time motion of the $q\bar{q}$ pair in the meson. They are required to be solutions of an eigenvalue problem — the bound-state equation; $M_{n\nu}$ being the mass of the corresponding meson:

$$\left\{\begin{array}{l}[2E_p-M_{n\nu}]\varphi^{+}_{n\nu}(p) = \displaystyle\int\frac{q^2dq}{(2\pi)^3}[T^{++}_{\nu}(p,q)\varphi^{+}_{n\nu}(q)+T^{+-}_{\nu}(p,q)\varphi^{-}_{n\nu}(q)]\\[2ex] [2E_p+M_{n\nu}]\varphi^{-}_{n\nu}(p) = \displaystyle\int\frac{q^2dq}{(2\pi)^3}[T^{-+}_{\nu}(p,q)\varphi^{+}_{n\nu}(q)+T^{--}_{\nu}(p,q)\varphi^{-}_{n\nu}(q)].\end{array}\right. \quad (6)$$

Eq. (6) can be alternatively derived using the Bethe–Salpeter approach to mesonic states [2], and thus a close connection between the Bethe–Salpeter and Hamiltonian approaches to the theory can be established, including graphical rules, which allow one to build the T-amplitudes in Eq. (6) using Feynman-like diagrams [4]. The Hamiltonian (1) takes now a diagonal form in terms of mesonic operators,

$$\hat{\mathscr{H}} = \sum_{n,\nu}M_{n\nu}m^{\dagger}_{n\nu}m_{n\nu}, \quad (7)$$

with corrections to the leading regime (7) suppressed by N_C. The first correction, of order $1/\sqrt{N_C}$, is responsible for mesonic decays [3]. Notice that, as soon as the mass-gap equation is solved, no new information, at least in the leading order in N_C, is needed to proceed beyond the BCS level and to introduce mesonic states. Numerical analysis of the mass-gap equation (3) for various confining potentials $V(r)$ demonstrated

existence of extra, "excited", solutions — the vacuum replicas [2, 5]. The existence of an infinite tower of such replicas for power-like confining potentials $V(r) \propto r^{\alpha}$ was proved analytically and verified numerically for all α's from the allowed region $0 \leq \alpha \leq 2$, as well as for D=2 and D=4 (D is the dimensionality of the space-time) [6]. It was argued in [6, 4] that the appearance of such replicas is a consequence of a very peculiar behaviour of the dressed quark dispersive law E_p in the infrared region and, therefore, is closely related to chiral symmetry breaking. A similar conclusion was also made in a different approach in [7]. It was demonstrated in [4] that, with the proper definition of the chiral angle, one encounters no problem with the imaginary mass of the pion in the replica vacua — all mesons build in replicas being normal hadronic states, but with a heavier mass as compared to mesons created in the ground-state vacuum. It is quite natural then to require that the information about vacuum replicas is transfered beyond the BCS level, the full Hamiltonian should contain the sum over the index $\mathcal{N}$ numerating the replicas,

$$\hat{\mathcal{H}} = \sum_{n,\nu,\mathcal{N}} M_{n\nu\mathcal{N}} m^{\dagger}_{n\nu\mathcal{N}} m_{n\nu\mathcal{N}}. \tag{8}$$

In conclusion let us notice that the analysis of the generalised Nambu-Jona-Lasinio theories performed above is insensitive to the dimensionality of the space-time, the Lorentz nature of confinement, and its explicit form. We argue that the existence of replicas is a rule rather than an exception for any confining quark kernel and we believe it is not an artifact of the instantaneous interquark interaction used in our analysis in order to bypass the problem of the relative time. An approach to replicas, as to local excitations, was suggested in [8] and an effective diagrammatic technique was derived in order to take into account the effect of replicas in hadronic reactions. Another important ingredient of the theory of vacuum replicas is a mechanism of excitation of replicas as global objects. This work is in progress now and will be reported elsewhere.

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